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THE DESIGN OF ANTENNA ARRAYS
FOR MAXIMUM SIGNAL TO NOISE RATIO

by

Jon W. Eberle

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THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
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THE DESIGN OF ANTENNA ARRAYS FOR MAXIMUM SIGNAL TO NOISE RATIO

INTRODUCTION

Recently there has been considerable thought given to the construction of some very large antennas for several diverse applications. The need first arose for large antennas in the field of radio astronomy, where an increase in antenna resolution was of prime concern. This has been accomplished using two different approaches. The first approach, and the most obvious means of increasing resolution, was to increase the antenna size. From this approach has resulted some of the largest antennas now in existence. However the radio astronomers also have devised several interferometry techniques whereby resolution can be obtained from smaller antennas with large spacings between the antennas, thus simulating large apertures. In many applications, the various types of interferometers accomplish the same result as one large antenna but with a significant reduction in cost.

For many present day applications, antennas having large gains as well as high resolutions are needed for such applications as long range radar, deep space communications and use of passive reflectors for communication between points on earth. In each of these applications, the greatest concern is to increase the system signal to noise ratio. This can be accomplished by increasing the signal, thus the desire for higher gain antennas and also by decreasing the noise level of the system, which is one of the reasons for the interest in masers and parametric amplifiers. There are many cases however where the addition of a low noise device would not affect an improvement in the overall signal noise ratio of the system. This situation would occur when the noise level in the system preceeding the amplifier would exceed the noise level of the amplifier itself. The purpose of this report is to investigate the noise level of various antenna arrays and to show that the performance of an array can be optimized by control of several parameters such as element diameter, number of elements, spacing between elements, feedline attenuation, etc.

In optimizing the performance of an array, some parameter has to be chosen that will indicate the performance of one array compared with another. Also a measure of the noise temperature of the array is needed in order to determine the improvement in overall system signal to noise ratio by the use of a low noise amplifier. For these purposes, the ratio of antenna gain to antenna noise temperature has been used. This quantity can be easily calculated knowing the various

parameters involved and hence will be used throughout this report as the performance figure for the various arrays to be considered.

The reason for considering arrays for use in high gain low noise applications is that it is believed that they offer several distinct advantages over single large apertures of comparable gain. These advantages can be divided into three groups, i.e., mechanical, electrical and cost. Under the mechanical heading are the items of antenna accuracy and antenna mount considerations. Although only parabolic elements are considered in this report, for reasons which will be given later, the same general conclusions will apply to other types of elements that might be used. At a fixed frequency, as gain is increased, the antenna size necessarily increases and the tolerances on the antenna necessary to realize the increase in gain remain the same. Thus up to a certain antenna size, no mechanical problems arise. However as the antenna size increases further, severe problems may arise in trying to maintain the tolerances. These problems could be alleviated by using smaller antennas in an array to achieve the desired gain level. Also as the antenna size increases, the antenna beamwidth decreases, which then requires high pointing accuracy from large antenna mounts, which is a condition that in many cases is not mechanically feasible. If an array type antenna were used, the mounts required to steer the array elements would be required to have a pointing accuracy proportional to the beamwidth of the elements, which could be appreciably less than that of the array. Thus higher tracking speeds could then be obtained if necessary, while maintaining sufficient accuracy in the mount.

In regard to the electrical advantages of an array, perhaps the most important one would be the ability to correct for atmosphere effects on the incoming wave front when used as a receiving antenna. When extremely high gain antennas are considered, the beamwidths of the antennas may be comparable with the magnitude of the disturbances occuring in the atmosphere. Movement of various masses of air with different water vapor content or density have the effect of making the atmosphere inhomogeneous which has various effects on a plane wave front, depending on the degree of inhomogenity. Under slightly turbulent conditions, the effect may be a shift in the direction of propagation with a plane wavefront being maintained. When highly turbulent conditions are present, it is possible that the wavefront may become crinkled and thus coherency would be lost over a single aperture. Under both of the above conditions, corrections could be incorporated into the array that would make a higher gain possible than could be obtained with a single antenna. In the case of slight turbulence,

where a wavefront changes its direction of propagation, the array could be electrically scanned to follow the scintillations in the wavefront without the elements having to be mechanically shifted as the beamwidth of the elements would be made larger than the maximum magnitude of the scintillations expected. This in fact would determine the maximum size of elements that should be used under such conditions. When highly turbulent conditions are present, a different approach would have to be used. If a wavefront existed such as shown in Fig. 1, the elements

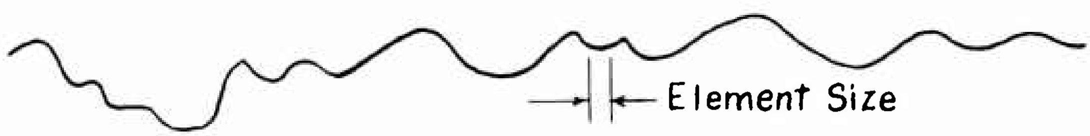

Fig. 1. Crinkled wavefront.

would have to be individually controlled and the various outputs combined coherently. Needless to say, this would represent a very undesirable condition. Another advantage of an array would be the ability to transmit large amounts of power by the use of smaller phase synchronized power sources located at each element or group of elements. This would alleviate the problems encountered when one high power unit would be used to feed an antenna. Also the problem of beamwidth control could be more easily solved with an array, where the problem of control would reduce to the problem of feeding the proper number of elements to achieve the desired beamwidth.

Where large highly accurate antennas are needed to achieve high gain, the cost advantage of an array can be appreciable compared with a single large antenna. This is true not only for the antenna structure itself but holds for the antenna mount also. Mounts designed to accurately steer a large antenna with an accuracy proportional to the beamwidth and at moderate tracking speeds can become exceedingly unwieldy structures that are quite expensive to construct.

As was stated previously, only parabolic reflector antennas will be considered in this report. The reasons for such a selection will be briefly outlined here. Among the various types of antennas that could be used as array elements, those belonging to the broadside radiating class offer the advantage that a higher gain can be realized with a less wieldy structure when compared with the endfire types. Among the broadside antennas, the reflector type has the advantage that they are easier to construct and lighter in weight and that they are an

inherently broadband device. Also by properly choosing the illumination of a reflector antenna, the noise power radiated into the feed from surrounding objects can be minimized. Thus a high gain low noise antenna can be realized, which fulfills the requirements for elements in an array used for high gain low noise applications.

ARRAY ANALYSIS

Before the array analysis is considered, a description of the types of arrays that will be dealt with will be given. The arrays to be considered here are of the corporate fed type. That is, the geometry of the feedlines in the various arrays to be considered is such that the physical length of the feedline from each element in the array to the output point is the same. Although the feeding of an array could be accomplished with much less feedline in certain cases, the corporate type feed has the advantage that the bandwidth of the array at broadside is considerably larger than could be obtained with other feedline geometries This results from the equal pathlengths from the input point to a wavefront propagating normal to the array aperture or visa versa Unfortunately this is not the case when the array is scanned from its broadside position. Under these conditions, the path lengths through the feedlines remain the same, but the distances from the elements to the wavefront no longer are equal and hence the bandwidth is decreased If bandwidth is to be preserved under such circumstances, additional phase shift is required in order to make the electrical pathlengths from the input point to the wavefront equal

It would be appropriate here to point out the various types of power dividers that are required to properly excite a corporate fed array. The type of power divider required, i. e., two way, three way, etc., is dependent upon the number of elements to be employed in the array. For example, considering line arrays, if two elements were desired, then a two way divider would be sufficient. However, if three elements were desired, a three way divider would be required as opposed to two, two way dividers If four elements were to be used, then three two way dividers would be adequate and etc. The effect of incorrectly splitting the power between the various elements would be to decrease the available gain in the receiving mode or possibly an undesirable aperture distribution in the transmitting mode. The effect of using the wrong type of divider is small when few elements are used and becomes progressively worse as the number of elements is increased

Both line and square arrays of varying number of elements and varying element sizes are to be considered here. The attenuation figure for the feedlines used in the calculations is 0.00012 nepers/foot at a wavelength of 6 inches (2000mc). In all cases, minimum spacings between the elements center to center is two element diameters, which permits scan angles up to 60° without aperture blockage from adjacent elements.

There are generally two factors that tend to degrade the performance of an array. The first of these is the attenuation occuring in the feedlines connecting the various elements. This factor affects the performance in two ways, first by decreasing the maximum available gain from the elements and second by increasing the amount of noise power in the antenna system, with the result that the ratio of gain to noise temperature, hereafter designated by η, is decreased. The second degradating factor for arrays would be the accuracy of spacing of the elements and the accuracy of excitation of the elements. It will be shown that the optimum form of array for maximum η is one having a small number of elements, and hence it is believed that the accuracy of spacing and excitation would not be a significant degradating factor in the cases considered here. For this reason it will not be considered further.

Line Arrays

Assume a line array as shown in Fig. 2 with the number of elements designated as n and the element diameters as d. For a

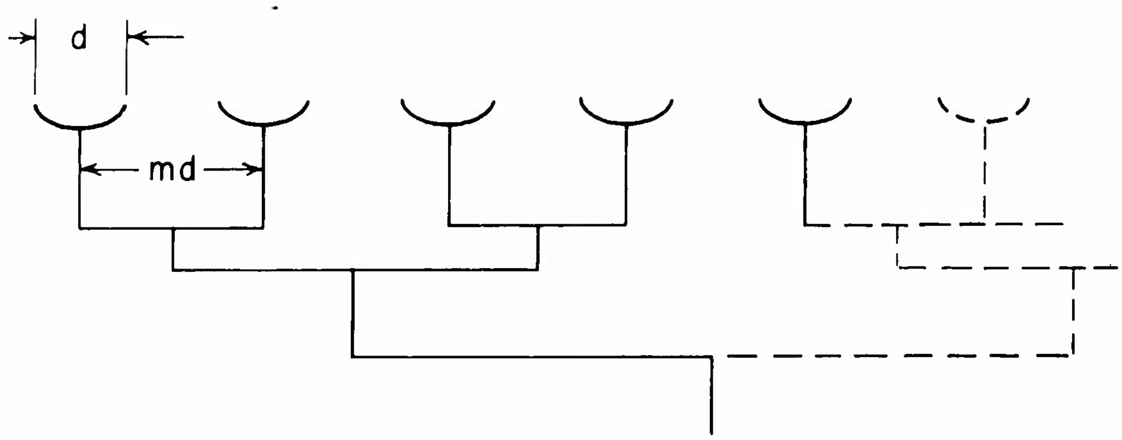

Fig. 2. Corporate fed line array.

corporate type feedline arrangement as shown in Fig. 2, the length of feedline from the input point of the array to any element, ℓ, is given by Eq. (1),

$$(1) \qquad \ell = \frac{m(n-1)d}{2}$$

where m is the center to center spacing of the array elements in terms of the element diameter. If the attenuation of the feedline is α nepers/foot, the total attenuation of the electric field induced in one antenna in propagating from the antenna to the feed point is $\alpha \ell$ nepers. If the contributions from each antenna are considered, the total gain of the array is

$$(2) \qquad G_{\substack{\text{Total}\\ \text{Line}}} = G_{\text{Element}} \sqrt{n}\, e^{-\alpha \ell} = \sqrt{6n}\, \frac{d}{\lambda}\, e^{\frac{-\alpha m(n-1)d}{2}}$$

where $\sqrt{6}\,\frac{d}{\lambda}$ [1] is the voltage gain over a $\frac{1}{2}\lambda$ dipole of a uniformly illuminated parabolic reflector. Equation (2) is plotted in Fig. 3 for various element diameters and numbers of elements. If maximum gain were desired for a given condition, the equation could be differentiated and the remaining parameters evaluated by equating the derivative to zero. As can be seen from Fig. 3, the gain curve reaches a maximum and then falls off. This is caused by rate of increase of feedline loss being greater than the rate of increase of antenna gain by the addition of more elements.

In the same way, the noise temperature in the system due to the attenuation in the feedlines increases as more elements are added. This noise temperature as well as the noise temperature of the antenna itself, resulting from the antenna beam being pointed toward regions of space or obstacles such as the ground, supporting structures, etc., which are above absolute zero, causes the antenna to have a noise temperature at the feed point.[2] If this temperature is appreciably higher than the temperature of the input stages of the amplifier connected to the antenna feed point, it will determine the noise level of the entire system and little would be gained by reducing the temperature of the input stages of the amplifier. Thus the antenna temperature, under certain conditions will determine the overall system noise level.

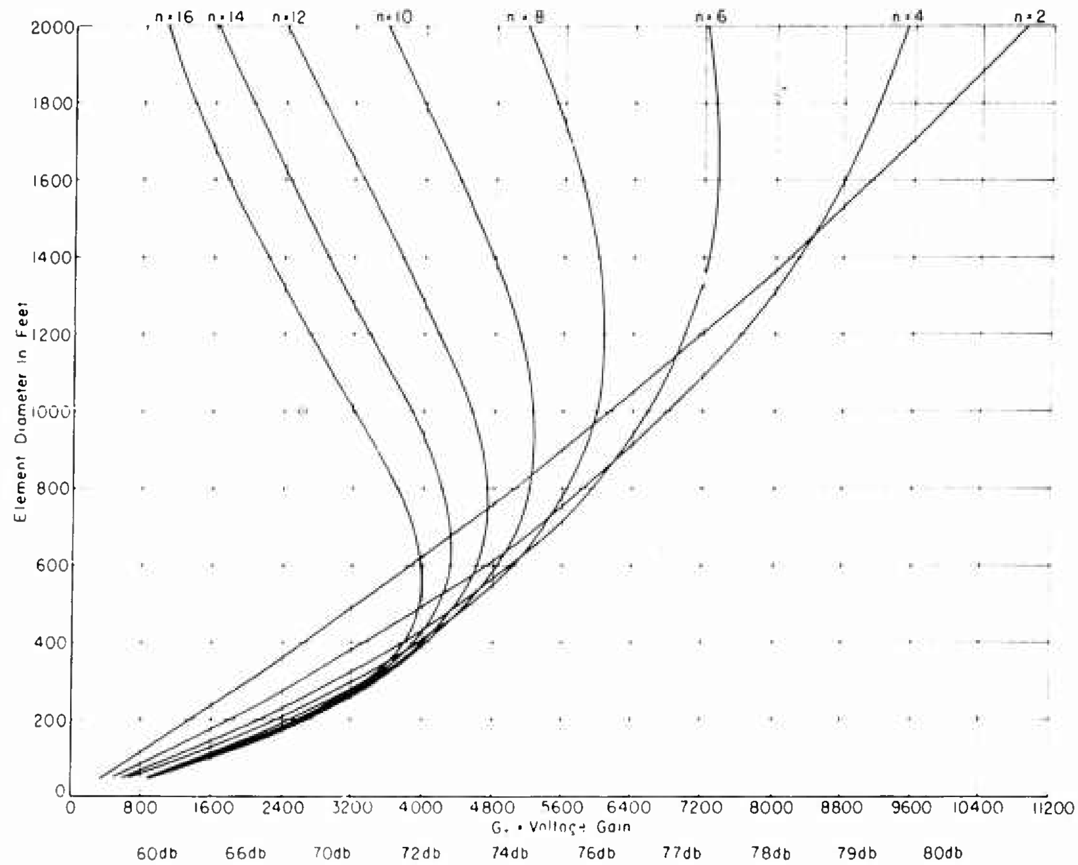

Fig. 3. Voltage gain vs element diameter for corporate fed line array m = 2, η = 12 x 10^{-5}.

Assuming the noise temperature of the elements alone, due to the temperature of regions in the antenna beam, to be T_A in degrees Kelvin and the temperature of the feedlines to be T_o °K, which is usually taken as ambient unless the lines are cooled, the total noise temperature at the antenna feed point is given by the equation

$$N_T = T_A e^{-2\alpha \ell} + T_o(1 - e^{-2\alpha \ell}) \tag{3}$$

where ℓ is given by Eq. (1). For a line array, the ratio η then becomes

$$\eta = \frac{G_{\substack{Total\\Line}}}{N_T} = \frac{\sqrt{6n}\,\frac{d}{\lambda}\, e^{\frac{-\alpha m(n-1)d}{2}}}{T_A e^{-\alpha m(n-1)d} + T_o(1 - e^{-\alpha m(n-1)d})} \tag{4}$$

This quantity is shown in Fig. 4 for the same element diameters and number of elements as was used in Fig. 3. It can be seen that in order to obtain the maximum value of η for a given number of elements, smaller diameters have to be used. The value of T_A in Fig. 4 was taken to be 10°K, which is a practical value if there are no high temperature sources such as the sun, radio stars, etc., in the antenna beam.

Square Array

A generalized square array is shown in Fig. 5. There are two possible methods for feeding such an array. In the first method, which requires a larger amount of feedline, the elements are connected as shown in Fig. 6a for a 4 element array. The second method, shown in Fig. 6b, connects the elements by radial feedlines which reduces the required feedline length by $1/\sqrt{2}$. The radial feeding is the type considered in the following discussion.

For the square array with radial corporate feeding, the length of feedline from an element to the feed point is given by

$$\ell = \sqrt{2}\, m\, (\sqrt{n} - 1)\, \frac{d}{2} \tag{5}$$

The gain for the square array then can be expressed as

$$G_{\substack{Total\\Square}} = \sqrt{6n}\,\frac{d}{\lambda}\, e^{\frac{-\alpha m(\sqrt{n} - 1)d}{\sqrt{2}}} \tag{6}$$

This expression is shown plotted for various numbers of elements and a range of element diameters in Fig. 7. The curves have the same general character as those in Fig. 3 for the line array, but do not fall off as rapidly in gain as the element diameter is decreased.

If again the noise temperature of the antennas and feedlines is considered, the noise temperature of the two is found to be

$$(7) \qquad N_T = T_A\, e^{-\sqrt{2}\alpha m(\sqrt{n}-1)d} + T_o\left(1 - e^{-\sqrt{2}\alpha m(\sqrt{n}-1)d}\right) .$$

The ratio η can then be found for the square array with the result

$$(8) \qquad \eta = \frac{G_{\substack{Total\\Square}}}{N_T} = \frac{\sqrt{6n}\,\frac{d}{\lambda}\, e^{\frac{-\alpha m(\sqrt{n}-1)d}{\sqrt{2}}}}{T_A e^{-\sqrt{2}\alpha m(\sqrt{n}-1)d} + T_o\left(1 - e^{-\sqrt{2}\alpha m(\sqrt{n}-1)d}\right)} .$$

The plot of this equation is shown in Fig 8(a). It is evident that the maximum value of η occurs when the smallest number of elements is used and when element sizes larger than 100 feet are used. When element sizes smaller than 100 feet are considered, it can be seen from Fig. 8(b) that the situation is reversed, where now the larger number of elements give the larger values of η.

Amplifiers in the Feedlines

In the light of the results obtained for the line and square arrays, it becomes apparent that the system signal to noise ratio or gain to noise temperature ratio cannot be increased any desired amount by increasing the array size Also it was found that the array gain could not be increased without bound by adding additional elements. In both cases, the feedline losses was the one factor that imposed the limitations. One obvious solution to th problem would be to decrease the feedline attenuation. Figure 9 shows the effect of such a decrease on the ratio η for a four element, radial corporate fed square array. As can be seen from the figure, this causes the curves to peak at larger values of element diameter. However it can be seen that a significant improvement can be obtained for all element sizes by decreasing the feedline attenuation.

In many cases, the feedline attenuation cannot be decreased appreciably and some other means for increasing the gain and η for an array is needed. At first thought, the addition of preamplifiers to the feedlines should effect an improvement in both. However upon

closer examination, it becomes clear that certain restrictions must be placed on the parameters of the preamplifier. For example, consider one section of an array with and without the preamplifiers present. These are shown in Figs. 10(a) and (b). For Fig. 10(a) the ratio η is

$$(9) \qquad \eta = \frac{G G_A S e^{-2\alpha \ell}}{G_A(T_A+T)e^{-2\alpha \ell} + T_o(1 - e^{-2\alpha \ell})}$$

and for Fig. 10(b), the ratio η is

$$(10) \qquad \eta = \frac{G S e^{-2\alpha \ell}}{T e^{-2\alpha \ell} + T_o(1 - e^{-2\alpha \ell})} \quad .$$

For an improvement in η to result from the addition of the preamplifier, the following inequality must hold

$$(11) \qquad \frac{G G_A S e^{-2\alpha \ell}}{G_A(T_A+T)e^{-2\alpha \ell} + T_o(1 - e^{-2\alpha \ell})} > \frac{G S e^{-2\alpha \ell}}{T e^{-2\alpha \ell} + T_o(1 - e^{-2\alpha \ell})} \quad .$$

This can be reduced to

$$(12) \qquad G_A T e^{-2\alpha \ell} + G_A T_o - G_A T_o e^{-2\alpha \ell} > G_A(T+T_A)e^{-2\alpha \ell} + T_o(1-e^{-2\alpha \ell}).$$

By rearranging and grouping, the following expression results

$$(13) \qquad T_A < \frac{T_o(G_A - 1)(1 - e^{-2\alpha \ell})}{G_A \, e^{-2\alpha \ell}} \quad .$$

When the amplifier gain is large, this can be reduced to

$$(14) \qquad T_A < \frac{T_o(1 - e^{-2\alpha \ell})}{e^{-2\alpha \ell}} \quad .$$

Thus the temperature of the input stages of the amplifier has to be less than the temperature contributed by the feedlines divided by the transmission coefficient of the transmission line having length ℓ and attenuation factor α. This condition effectively states that the preamplifier has to amplify the level of the antenna noise so that the increase in noise due to the feedline losses is a small percentage of the total noise present at the antenna terminals. In Fig. 11 is shown the ratio

$$\frac{T_o(1 - e^{-2\alpha\ell})}{e^{-2\alpha\ell}}$$

as a function of $e^{-2\alpha\ell}$. Thus depending upon the transmission coefficient of the length of transmission line between one of the elements and the feed point of the array, the advantage of using a preamplifier or not can be determined. For example, consider the square array having radial feedlines and four elements. The transmission coefficient of the feedlines vary from .99 for the smaller elements to around .5 for the largest elements, which from Fig. 11 determines that the temperature of the input stages of the preamplifier should be less than 300^{o}K for the largest elements to less than 20^{o}K for the smallest elements. Thus a parametric amplifier could be used with the large elements whereas a maser would be required for the smaller elements.

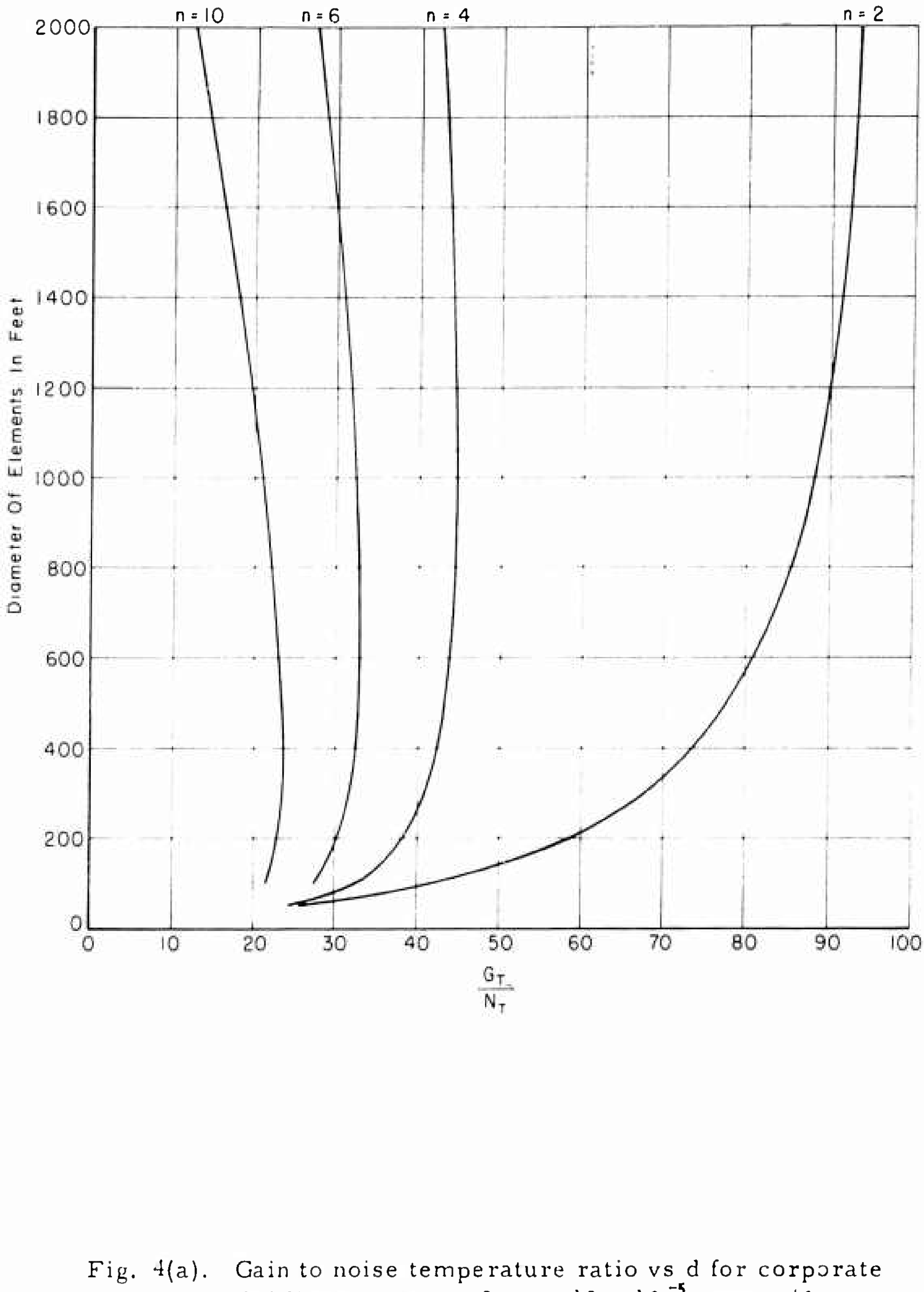

Fig. 4(a). Gain to noise temperature ratio vs d for corporate fed line array m = 2, α = 12 x 10^{-5} nepers/foot.

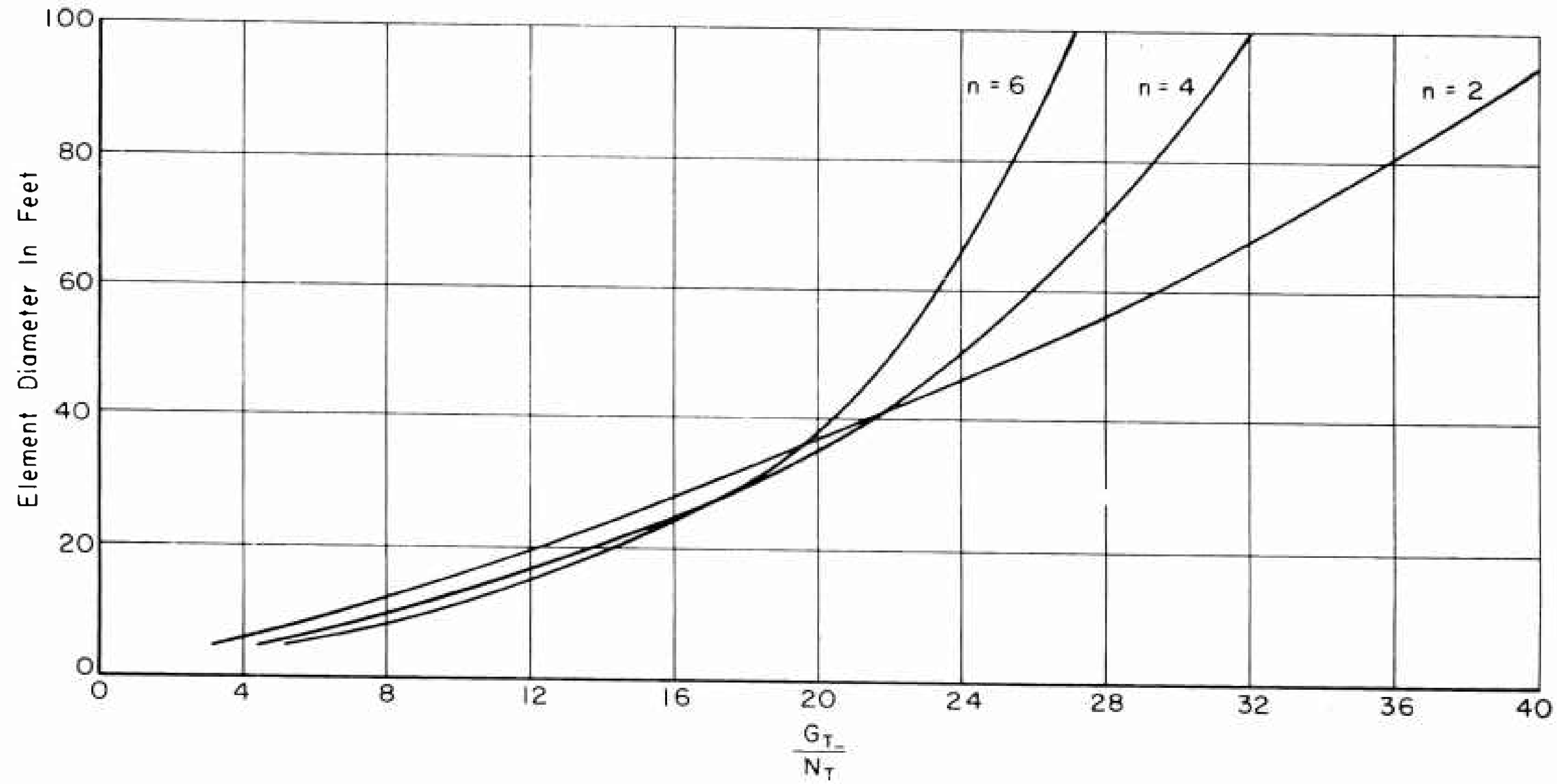

Fig. 4(b). Gain to noise temperature ratio vs d for corporate fed line array m = 2, α = 12 x 10^{-5} nepers/foot.

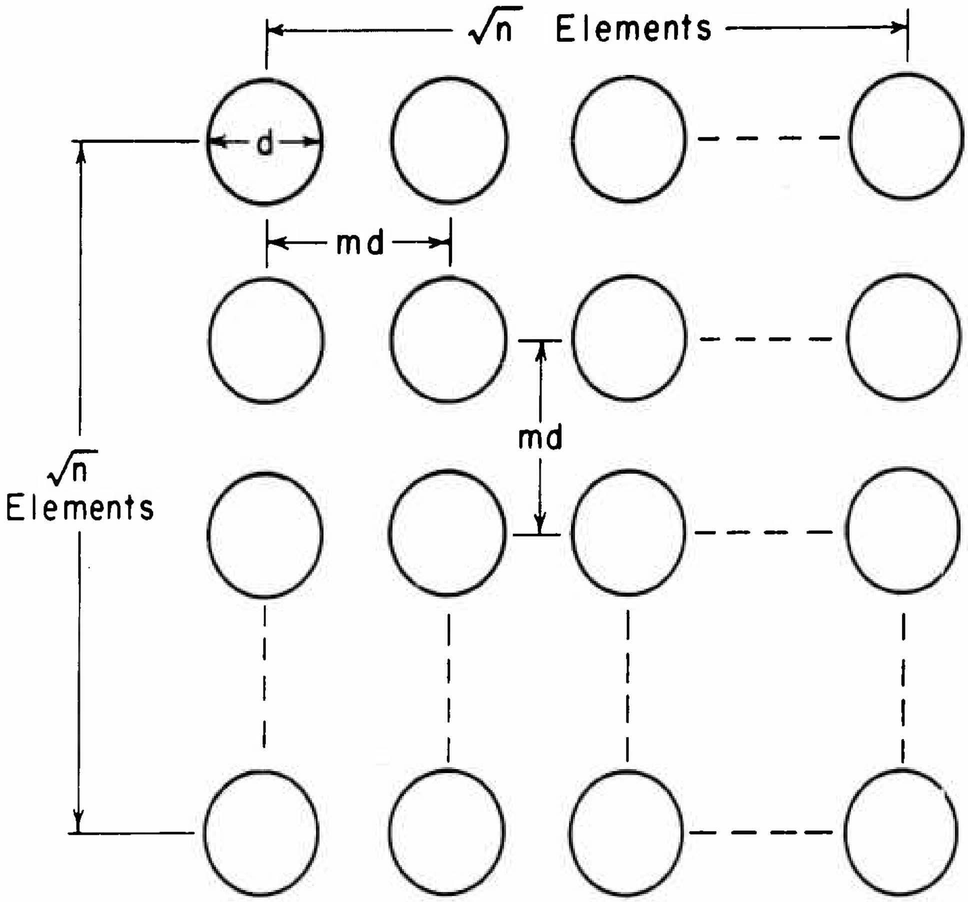

Fig. 5. Square array, n elements.

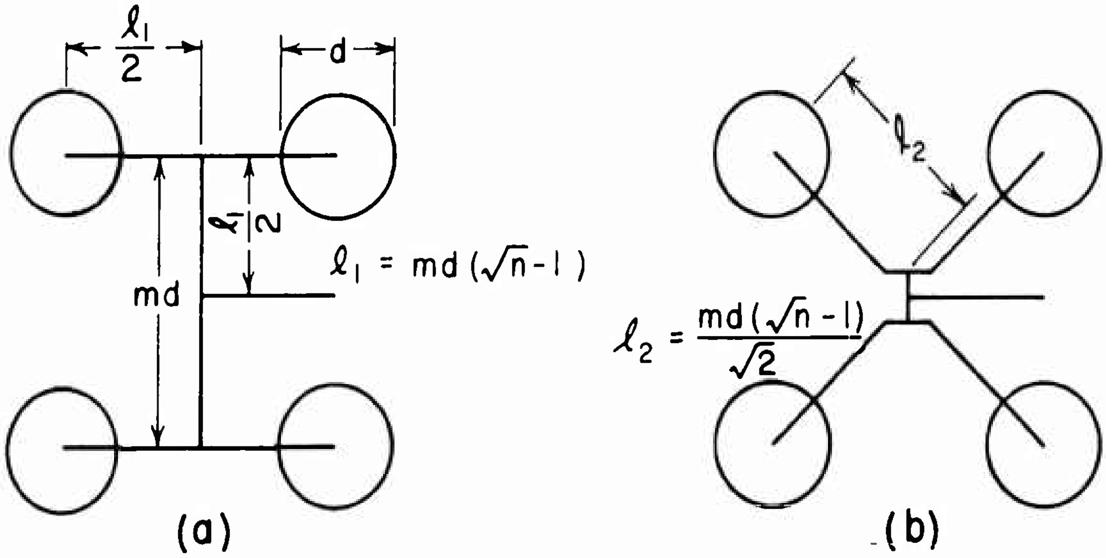

Fig. 6. Corporate fed 2 x 2 arrays.

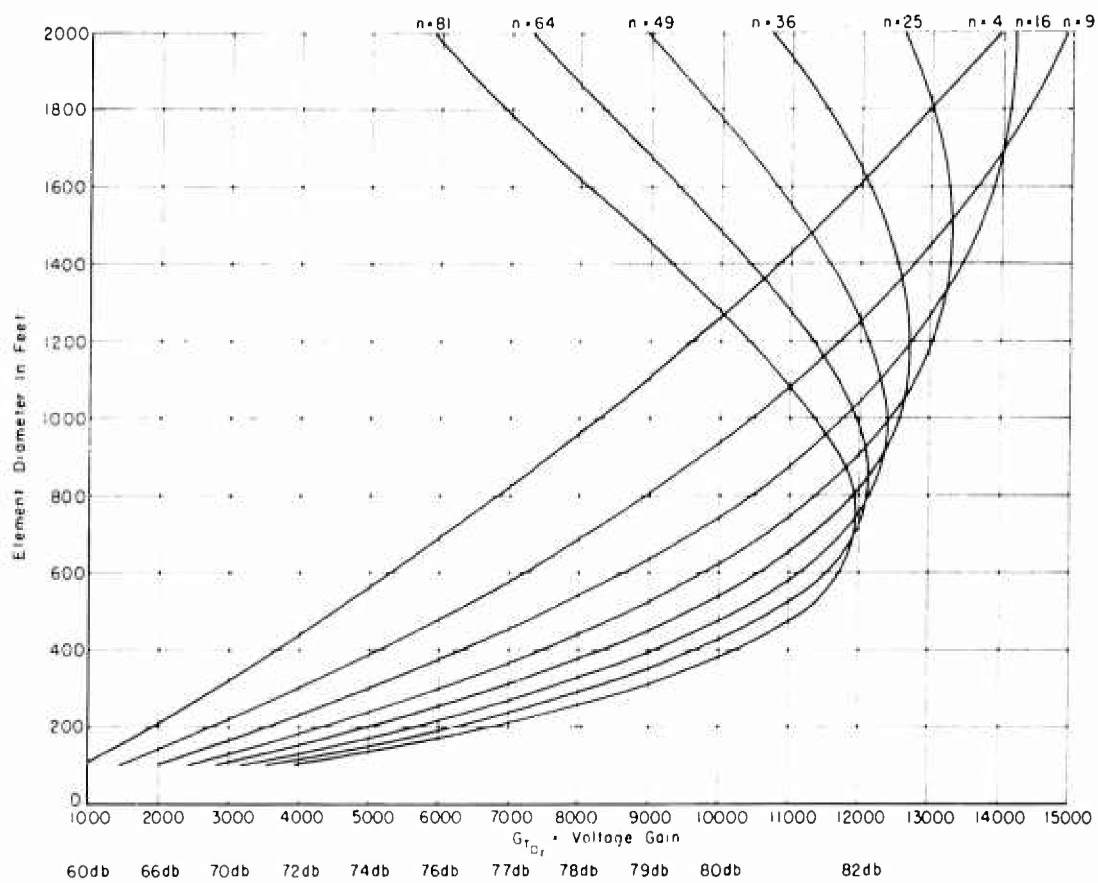

Fig. 7. Voltage gain vs element diameter for square radial corporate fed array m = 2, $\alpha = 12 \times 10^{-5}$ nepers/foot.

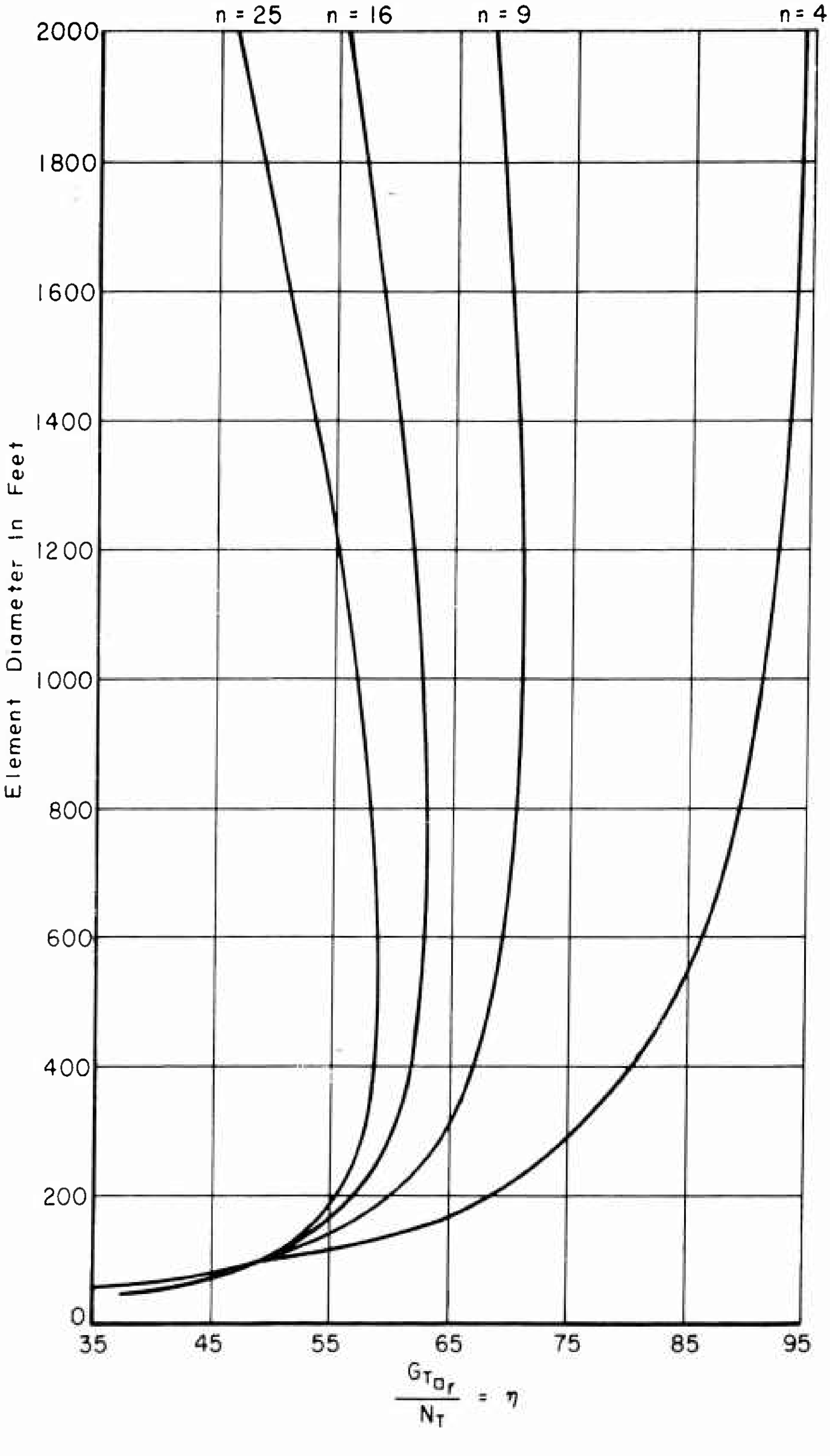

Fig. 8(a). Gain to noise temperature ratio vs element diameter for radial corporate fed square array. $m = 2$, $\alpha = 12 \times 10^{-5}$ nepers/foot.

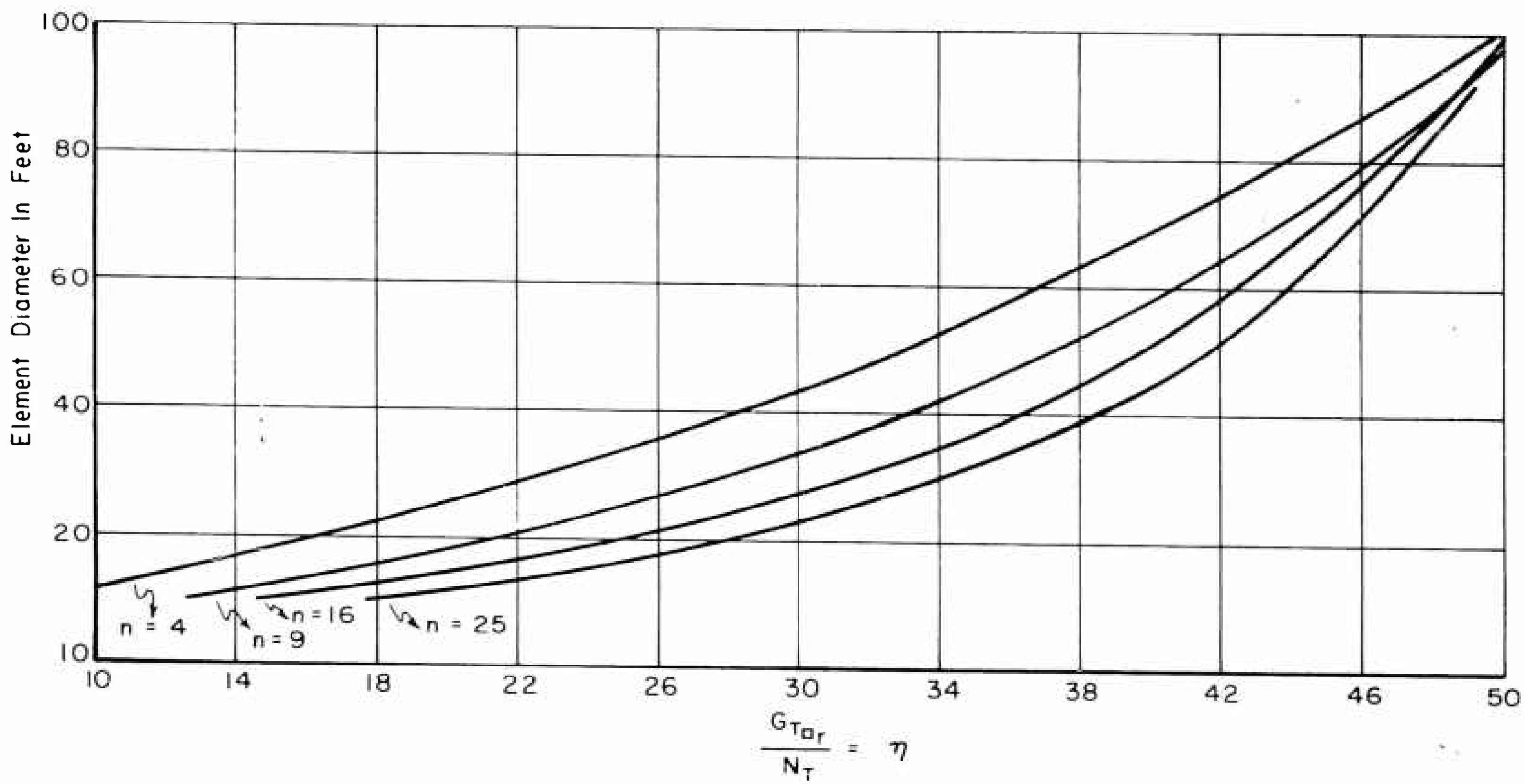

Fig. 8(b). Gain to noise temperature ratio vs element diameter for radial corporate fed square array. $m = 2$, $\alpha = 12 \times 10^{-5}$ nepers/foot.

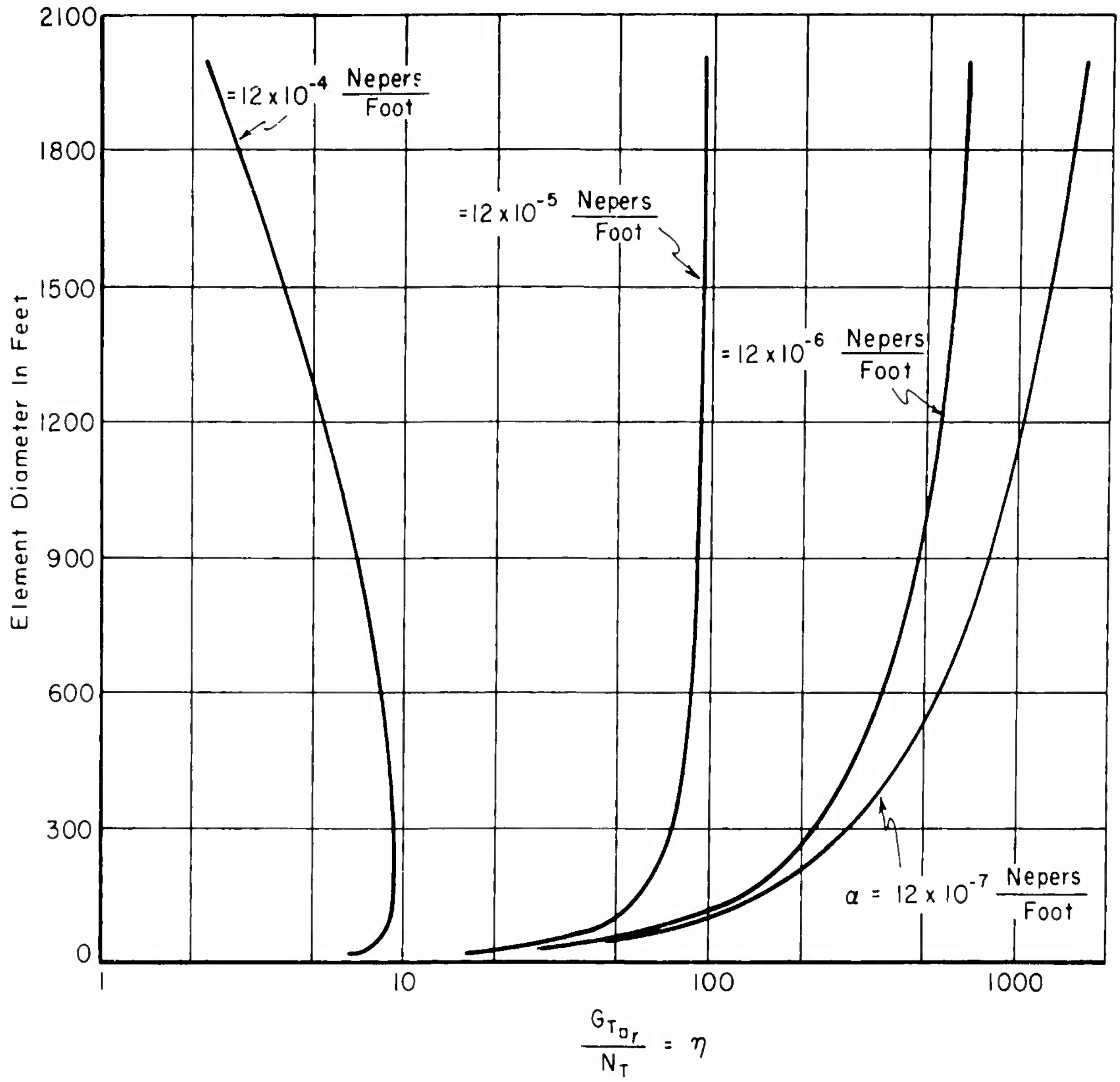

Fig. 9. Gain to noise temperature ratio vs element diameter for a four element square array m = 2.

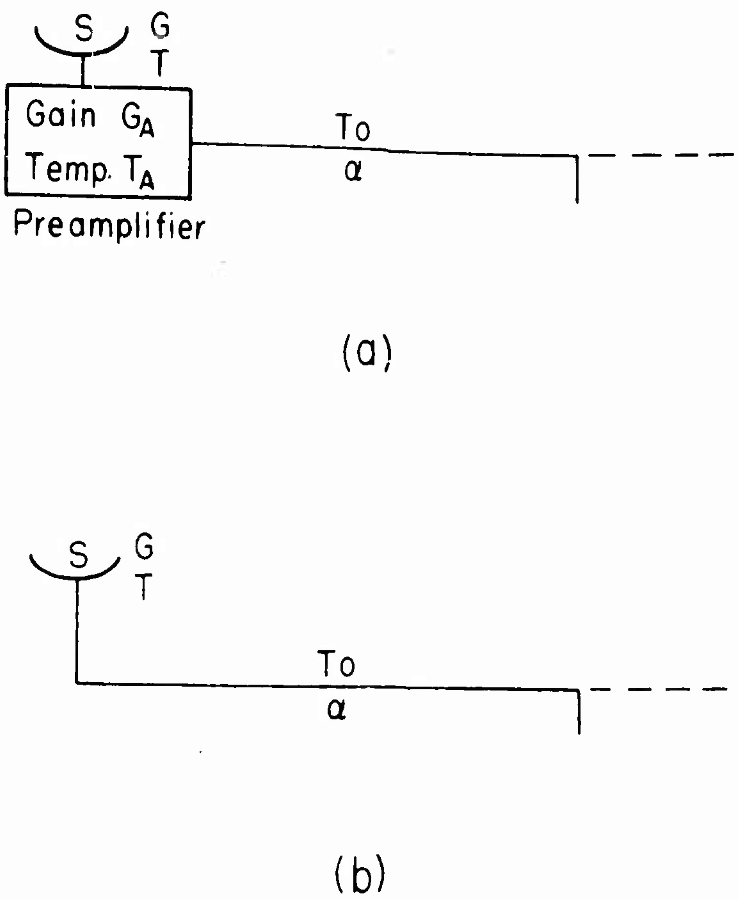

Fig. 10. Amplifiers in feedlines.

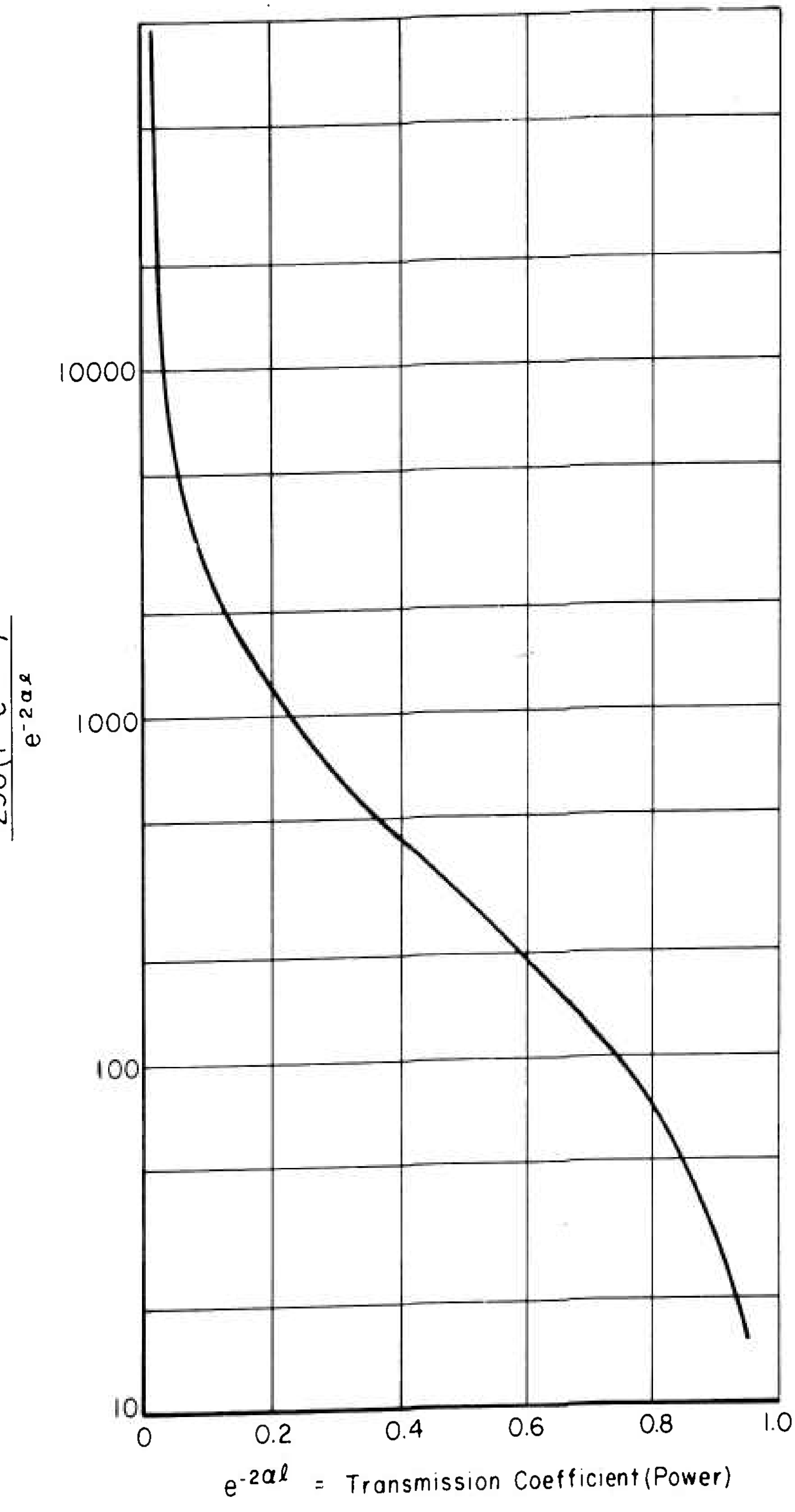

Fig. 11. $\frac{T_o(1-e^{-2\alpha\ell})}{e^{-2\alpha\ell}}$ vs $e^{-2\alpha\ell}$.

CONCLUSIONS

The optimum configuration of an array type antenna consists of a small number of large elements as opposed to a large number of small elements when both high gain or a large ratio of antenna gain to noise temperature of the system is desired.

The condition for maximum gain does not coincide with the conditions for maximum values of η. When maximum η is desired, a smaller element diameter can be used contrasted with the conditions for maximum gain

Much can be gained by reducing the attenuation figure of the waveguide feedlines This not only increases gain but reduces noise so that a gain is made in both respects.

Depending upon the attenuation factor of the waveguide feedlines, some improvement can be obtained by the addition of preamplifiers to the elements. The temperature of the input stages of the preamplifier is determined by the temperature of the transmission lines and by the attenuation factor of the feedlines, in order that an improvement be realized.

Reducing the physical temperature of the feedlines will result in an improvement in the value of η and depending on the amount of cooling introduced, the gain may also increase. This would occur when a superconductive state would be reached in the feedlines.

It is believed that the array type antenna offers certain advantages over the single aperture antenna when such factors as high power transmission, correction for atmospheric effects are needed, when extremely high gain is needed or where bandwidth control is desired.

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2. Hame, T. G., Eberle, J. W., Caldecott, R., and Taylor, R. C., "High Gain Low Noise Antenna Research," Tenth Annual Antenna Symposium, University of Illinois, October 1960.

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